Problem 1. Use integration by parts to find $\int \log x \, dx$.

Solution. We will use integration by parts to do it. All we have to do is to choose u and dv. We take

$$\begin{cases} u = \log x \\ dv = dx \end{cases} \tag{1}$$

Now, differentiating and integrating respectively these two equations we obtain

$$\begin{cases} du = \frac{dx}{x} \\ v = x \end{cases} \tag{2}$$

The integration by parts formula tells us that

$$\int u \, dv = uv - \int v \, du + C \tag{3}$$

so we replace what we got in (1) and (2) in this last equation to get

$$\int \log \, dx = x \log x - \int x \, \frac{dx}{x} + C = x \log x - \int dx + C = x \log x - x + C$$

So if we factor we finally obtain

$$\int \log x \, dx = x(\log x - 1) + C$$

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